ICC 2014 Tutorial
Optimal Resource Management for Future Cellular and Heterogeneous Networks

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14 June 2014
Biography: Emil Björnson

- 1983: Born in Malmö, Sweden

- 2007: Master of Science in Engineering Mathematics, Lund University, Sweden

- 2011: PhD in Telecommunications, KTH, Stockholm, Sweden
  Advisors: Björn Ottersten, Mats Bengtsson


- 2014: Assistant Professor in Communication Systems, Linköping University, Sweden
Biography: Eduard Jorswieck

• 1975: Born in Berlin, Germany

• 2000: Dipl.-Ing. in Electrical Engineering and Computer Science, TU Berlin, Germany

• 2004: PhD in Electrical Engineering, TU Berlin, Germany
  Advisor: Holger Boche

• 2006: Post-Doc Fellowship and Assistant Professorship at KTH, Stockholm, Sweden

• 2008: Full Professor and Head of Chair of Communications Theory at TU Dresden, Germany
Book Reference

- Tutorial is Partially based on a Book:

**Optimal Resource Allocation in Coordinated Multi-Cell Systems**

*Research book by E. Björnson and E. Jorswieck*

*Foundations and Trends in Communications and Information Theory, Vol. 9, No. 2-3, pp. 113-381, 2013*

- 270 pages
- E-book for free ([from our homepages](http://flexible-radio.com/emil-bjornson))
- Matlab code is available online
General Outline

- Introduction
  - Problem formulation and general system model

- Optimal Single-Objective Resource Management
  - Which problems are practically solvable and why?

- Multi-Objective Network Optimization
  - Definitions and potential applications

- Applications in Future Networks
  - Energy-Efficiency in Multi-User Multi-Hop Networks
  - Distributed Implementation and Spectrum Sharing
  - Resource Management with Confidentiality Constraints

Fundamentals
Covered by Emil Björnson in first 90 min

New Applications
Covered by Eduard Jorswieck in second 90 min
Part 1

Fundamentals
Outline: Part 1 – Fundamentals

• Introduction
  - Multi-cell structure, system model, performance measure

• Problem Formulation
  - Resource management: Multi-objective optimization problem

• Subjective Resource Management
  - Utility functions, different computational complexity

• Multi-Objective Network Optimization
  - What are the future potentials of this theory?
Section

Introduction
Introduction

• Wireless Connectivity
  - A natural part of our lives

- Text messages
- Social networks
- Web browsing
- Voice call
- Video streaming
- Gaming

• Rapid Network Traffic Growth
  - 61% annual data traffic growth
  - Exponential increase!
  - Extrapolation: 20x until 2020
    200x until 2025
    2000x until 2030

Exabytes per Month

Source: Cisco VNI Mobile, 2014
Introduction: Evolving Networks for Higher Traffic

- Increasing Network Capacity \([\text{bit/s/Area}]\)
  - Consider a given area
  - How to handle 1000x more data traffic?

- Formula for Network Capacity:

\[
\text{Capacity [bit/s in area]} = \text{Spectral efficiency [bit/s/Hz/Cell]} \cdot \frac{\text{Cell density [Cell/Area]}}{\text{Available spectrum [in Hz]}}
\]

### History (1965-2010)

<table>
<thead>
<tr>
<th></th>
<th>Spectral efficiency</th>
<th>Cell density</th>
<th>Spectrum</th>
<th>Total</th>
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<tbody>
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<td>History (1965-2010)</td>
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<td>1600x</td>
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<td>Future: Nokia</td>
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<td>Future: SK Telecom</td>
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Considered in this tutorial

Björnson & Jorswieck: Optimal Resource Management
• Problem Formulation (vaguely):
  - Transfer information wirelessly to users
  - Divide radio resources among users (time, frequency, space)

• Downlink Cellular System
  - Many transmitting base stations (BSs)
  - Many receiving users
  - Multiple-input multiple-output (MIMO)

• Sharing a Frequency Band
  - All signals reach everyone!

• Limiting Factor
  - Inter-user interference
Introduction: Multi-Antenna Transmission

• Traditional Ways to Manage Interference
  - Avoid and suppress in time and frequency domain
  - Results in orthogonal access techniques: TDMA, OFDMA, etc.

• Multi-Antenna Transmission
  - Beamforming: Spatially directed signals
  - Adaptive control of interference
  - Serve multiple users: Space-division multiple access (SDMA)

Main difference from classical resource management!
Introduction: From Single-Cell to Multi-Cell

- Naïve Multi-Cell Extension
  - Divide BS into disjoint clusters
  - SDMA within each cluster
  - Avoid inter-cluster interference
  - Fractional frequency-reuse

- Coordinated Multi-Cell Transmission
  - SDMA in multi-cell: All BSs cooperate, no fixed clusters
  - Frequency-reuse one: Interference managed by beamforming
  - Many names: co-processing, coordinated multi-point (CoMP), network MIMO, multi-cell processing
  - Implementation: Artemis pCell (?)

- Almost as One Super-Cell
  - But: Different data knowledge, channel knowledge, power constraints!
Basic Multi-Cell Coordination Structure

- General User-Centric Multi-Cell Coordination
  - Adjacent base stations coordinate interference
  - Some users served by multiple base stations

Dynamic Cooperation Clusters

- Inner Circle $\mathcal{D}_k$: Serve users with data
- Outer Circle $\mathcal{C}_k$: Suppress interference
- Outside Circles:
  - Negligible impact
  - Impractical to acquire information
  - Difficult to coordinate decisions

Example: Ideal Joint Transmission

- All Base Stations Serve All Users Jointly = One Super Cell

The whole network:

\[ C_k = D_k = \{1, \ldots, K_r\} \]
Example: Wyner Model

- Abstraction: User receives signals from own and neighboring base stations

- One or Two Dimensional Versions

- Joint Transmission or Coordination between Cells
Example: Coordinated Beamforming

- One Base Station Serves Each User
- Interference Coordination Across Cells

Special Case

Interference channel
Example: Soft-Cell Coordination

- Heterogeneous Deployment
  - Conventional macro BS overlaid by short-distance small BSs
  - Interference coordination and joint transmission between layers
Example: Cognitive Radio

- Secondary System Borrows Spectrum of Primary System
  - Underlay: Interference limits for primary users

Other Examples

- Spectrum sharing between operators
- Physical layer security
Optimizing Resource Management: First Definition

- **Problem Formulation (imprecise):**
  - Select beamforming to maximize “system utility”
  - Means: Allocate power to users and in spatial dimensions
  - Satisfy: Physical, regulatory & economic constraints

- **Some Assumptions:**
  - Linear transmission and reception
  - Perfect synchronization (whenever needed)
  - Flat-fading channels (e.g., using OFDM)
  - Centralized optimization (e.g., cloud-RAN) Part 2: Distributed implementation
  - Cooperation clusters are given
  - Perfect channel knowledge
  - Ideal transceiver hardware

Can be relaxed: See our book
Multi-Cell System Model

- $K_r$ Users: Channel vector $\mathbf{h}_k = [\mathbf{h}_{1k}^T \ldots \mathbf{h}_{K_r k}^T]^T$ to User $k$ from all BSs
- $N_j$ Antennas at $j$th BS (dimension of $\mathbf{h}_{jk}$) – small or large
- $N = \sum_j N_j$ Antennas in Total (dimension of $\mathbf{h}_k$)

![Diagram showing the system model](attachment:image.png)

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**Intended Information Symbols**
**Beamforming** ($\mathbf{D}_k$ describes inner circle)
**Channels** ($\mathbf{C}_k$ describes outer circle)
**Noise** (and Distant Interference)
**Received Signals at Users**

---

**One System Model for All Multi-Cell Scenarios!**
Multi-Cell System Model: Dynamic Cooperation Clusters

• How are $D_k$ and $C_k$ Defined?
  - Consider User $k$:

  $$D_k = \begin{bmatrix}
  D_{1k} & 0 \\
  \vdots & \ddots \\
  0 & D_{K_k,k}
  \end{bmatrix}$$

  where $D_{jk} = \begin{cases}
  I_{N_j}, & \text{if } k \in D_j, \\
  0_{N_j}, & \text{otherwise},
  \end{cases}$

  $$C_k = \begin{bmatrix}
  C_{1k} & 0 \\
  \vdots & \ddots \\
  0 & C_{K_k,k}
  \end{bmatrix}$$

  where $C_{jk} = \begin{cases}
  I_{N_j}, & \text{if } k \in C_j, \\
  0_{N_j}, & \text{otherwise}.
  \end{cases}$

• Interpretation:
  - Block-diagonal matrices
  - $D_k$ has identity matrices for BSs that send data
  - $C_k$ has identity matrices for BSs that can/should coordinate interference
• Example: Coordinated Beamforming
  - This is User $k$
  - Beamforming: $\mathbf{D}_k \mathbf{v}_k$
    Data only from BS$_1$:

$$
\mathbf{D}_k = \begin{bmatrix}
\mathbf{I}_{N_1} & 0_{N_2} & 0_{N_3} & 0_{N_4}
\end{bmatrix}
$$

- Effective channel: $\mathbf{c}_k \mathbf{h}_k$
  Interference from all BSs:

$$
\mathbf{C}_k = \begin{bmatrix}
\mathbf{I}_{N_1} & \mathbf{I}_{N_2} & \mathbf{I}_{N_3} & \mathbf{I}_{N_4}
\end{bmatrix}
$$
Multi-Cell System Model: Power Constraints

- Need for Power Constraints
  - Limit radiated power according to regulations
  - Protect dynamic range of amplifiers
  - Manage cost of energy expenditure
  - Control interference to certain users

- $L$ General Power Constraints:

$$
\sum_{k=1}^{K_r} v_k^H Q_{lk} v_k \leq q_l, \quad l = 1, \ldots, L
$$

- Weighting matrix (Positive semi-definite)
- Limit (Positive scalar)

All at the same time!
Multi-Cell System Model: Power Constraints (2)

- Recall: \[ \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \]

- Example 1, Total Power Constraint: \( L = 1: \) \( \mathbf{Q}_{1k} = \mathbf{I}_N \)
  \[ q_1 = \text{Maximal total power} \]

- Example 2, Per-Antenna Constraints:
  \( L = N: \) \( \mathbf{Q}_{1k} = \text{diag}(1, 0, \ldots, 0), \ldots, \mathbf{Q}_{Nk} = \text{diag}(0, \ldots, 0, 1) \)
  \[ q_l = \text{Maximal power at } l\text{th antenna} \]
Introduction: How to Measure User Performance?

- **Mean Square Error (MSE)**
  - Difference: transmitted and received signal
  - Easy to Analyze
  - Far from User Perspective?

- **Bit/Symbol Error Probability (BEP/SEP)**
  - Probability of error (for given data rate)
  - Intuitive interpretation
  - Complicated & ignores channel coding

- **Information Rate**
  - Bits per “channel use”
  - Mutual information: perfect and long coding
  - Anyway closest to reality?

All improves with SINR:

\[
\text{Signal} \quad \frac{\text{Interference + Noise}}{}
\]
Introduction: Generic Measure User Performance

• Generic Model
  - Any function of signal-to-interference-and-noise ratio (SINR):

  \[ g_k(\text{SINR}_k) = g_k \left( \frac{|h_k^H C_k D_k v_k|^2}{\sigma_k^2 + \sum_{i \neq k} |h_k^H C_k D_i v_i|^2} \right) \]

  for User \( k \)

  - Increasing and continuous function
  - For simplicity: \( g_k(0) = 0 \)

• Simple Examples:
  - Information rate: \( g_k(\text{SINR}_k) = \log_2(1 + \text{SINR}_k) \)
  - MSE: \( g_k(\text{SINR}_k) = \frac{\text{SINR}_k}{1 + \text{SINR}_k} \)

• Complicated Function
  - Depends on all beamforming vectors \( v_1, ..., v_{Kr} \)
Section: Introduction

Questions?
Section

Problem Formulation
General Formulation of Optimal Resource Management:

\[
\begin{align*}
\text{maximize} & \quad \{g_1(\text{SINR}_1), \ldots, g_{K_r}(\text{SINR}_{K_r})\} \\
\text{subject to} & \quad \sum_{k=1}^{K_r} v_k^H Q_{lk} v_k \leq q_l \quad \forall l.
\end{align*}
\]

Multi-Objective Optimization Problem
- Generally impossible to maximize for all users!
- Must divide power and cause inter-user interference
Performance Region

- Definition: Achievable Performance Region $\mathcal{R}$
  - Contains all feasible combinations $\{g_1(\text{SINR}_1), \ldots, g_K(\text{SINR}_K)\}$
  - Feasible = Achieved by some $\{v_1, \ldots, v_K\}$ under power constraints

Balance between users
Part of interest: Pareto boundary
Care about user 2
Care about user 1

Pareto Boundary
Cannot improve for any user without degrading for other users

Other Names
Rate Region
Capacity Region
MSE Region, etc.
Can the region have any shape?

No! Can prove that:
- Compact set
- Normal set

Upper corner in region, everything inside region
Performance Region (3)

- Some Possible Shapes

![Diagram showing different shapes of performance regions for weak and strong user-coupling.](image)

**User-Coupling**

Weak: Convex boundary
Strong: Concave boundary
Which Pareto Optimal Point to Choose?
- Tradeoff: Aggregate performance vs. fairness

Performance Region (4)

Utilitarian point
(Max sum performance)

Egalitarian point
(Max fairness)

Single user point

No Objective Answer

Utopia point outside of region
Only subjective answers exist!
Section: Problem Formulation

Questions?
Section

Subjective Resource Management
Subjective Approach

- System Designer Selects Utility Function $f : \mathcal{R} \rightarrow \mathbb{R}$
  - Describes subjective preference
  - Increasing and continuous function

Examples:

- Sum performance: $f(g) = \sum_k g_k$
- Proportional fairness: $f(g) = \prod_k g_k$
- Harmonic mean: $f(g) = K_R \left( \sum_k g_k^{-1} \right)^{-1}$
- Max-min fairness: $f(g) = \min_k g_k$

Known as A Priori Approach

Select utility function before optimization

Put different weights to move between extremes
Subjective Approach (2)

• Utility Function gives Single-Objective Optimization Problem:

\[
\begin{align*}
\text{maximize} & \quad f(g) \\
\text{subject to} & \quad \sum_{k=1}^{K_r} v_k^H Q_{lk} v_k \leq q_l \quad \forall l.
\end{align*}
\]

• This is the Starting Point of Many Researchers
  - Although selection of $f$ is inherently subjective
  - Affects the solvability
  - Should always have a motivation in mind!

**Pragmatic Approach**

Try to Select Utility Function to Enable Efficient Optimization
Complexity of Single-Objective Optimization Problems

• Classes of Optimization Problems
  - Different scaling with number of parameters and constraints

• Main Classes
  - Convex: Polynomial time solution
  - Monotonic: Exponential time solution
  - Arbitrary: More than exponential time

Practically solvable
Approximations needed
Hard to even approximate
What is a Convex Problem?
- Recall definitions:

**Convex Function**

For any two points on the graph of the function, the line between the points is above the graph.

**Examples:** $x^2$, $e^x$, $-\log_2(x)$

**Convex Problem**

\[
\begin{align*}
\text{minimize} & \quad f_0(x) \\
\text{subject to} & \quad f_m(x) \leq 0 \quad m = 1, \ldots, M
\end{align*}
\]

Convex if objective $f_0$ and constraints $f_1, \ldots, f_M$ are convex functions.
When is the Resource Management a Convex Problem?

- Original problem:

\[
\text{maximize } f(g) \quad \text{subject to } \sum_{k=1}^{K_r} v_k^H Q_{lk} v_k \leq q_l \quad \forall l.
\]

- Rewritten problem (replace SINR\(_k\) with variable \(\gamma_k\)):

\[
\text{minimize } f\left(g_1(\gamma_1), \ldots, g_{K_r}(\gamma_{K_r})\right) \quad \text{subject to } \left| h_k^H C_k D_k v_k \right|^2 \geq \gamma_k \left( \sigma_k^2 + \sum_{i \neq k} \left| h_k^H C_k D_i v_i \right|^2 \right) \quad \forall k,
\]

\[
\sum_{k=1}^{K_r} v_k^H Q_{lk} v_k \leq q_l \quad \forall l.
\]
Classification of Resource Management Problems

- Classification of Three Important Problems
  - The “Easy” problem
  - Weighted max-min fairness
  - Weighted sum performance

- We will see: These have Different Complexities
  - Difficulty: Too many spatial degrees of freedom
  - Convex problem only if search space is particularly limited
  - Monotonic problem in general
Complexity Example 1: The “Easy” Problem

- Given Any Point \((\tilde{g}_1, \ldots, \tilde{g}_{K_r})\) or SINRs \((\gamma_1, \ldots, \gamma_{K_r})\)
  - Find beamforming \(v_1, \ldots, v_{K_r}\) that attains this point
  - Fixed SINRs make the constraints convex:

\[
|h_k^H C_k D_k v_k|^2 \geq \gamma_k \left( \sigma_k^2 + \sum_{i \neq k} |h_k^H C_k D_i v_i|^2 \right)
\]

- Global solution in polynomial time – use CVX, Yalmip

**Total Power Constraints**

**Per-Antenna Constraints**

**General Constraints**
### Complexity Example 2: Max-Min Fairness

- **How to Classify Weighted Max-Min Fairness?**

\[
\text{maximize } f(g) = \min_k w_k g_k \quad \text{subject to } \sum_{k=1}^{K_r} v_k^H Q_{lk} v_k \leq q_l \quad \forall l.
\]

- Property: Solution makes \( w_k g_k \) the same for all \( k \)

---

**Bisection Algorithm**

1. Find start interval
2. Solve the “easy” problem at midpoint
3. If feasible:
   - Remove lower half
   - Else: Remove upper half
4. Iterate
Complexity Example 2: Max-Min Fairness (2)

- Solution: Simple Iterative Algorithm
  - Subproblem: Convex “Easy” problem
  - Few iterations: Linear convergence, one dimension (for any #users)

- Classification of Weighted Max-Min Fairness:
  - **Quasi-convex problem** (belongs to convex class)
  - Polynomial complexity in #users, #antennas, #constraints
  - Complexity might be feasible in practice

**Early work**

**Main references**

**Channel uncertainty**
How to Classify Weighted Sum Performance?

\[
\text{maximize } f(g) = \sum_{k=1}^{K_r} w_k g_k \quad \text{subject to } \sum_{k=1}^{K_r} v_k^H Q_{lk} v_k \leq q_l \quad \forall l.
\]

- Geometrically: \( w_1 g_1 + w_2 g_2 = \text{optimal-value is a line} \)

Optimal value is unknown!

- Distance from origin is unknown
- Line \( \rightarrow \) Hyperplane (dim: \#user - 1)
- Harder than max-min fairness
- Non-convex problem
Complexity Example 3: Weighted Sum Performance (2)

- Classification of Weighted Sum Performance:
  - Non-convex problem (despite convex power constraints)
  - Utility: Monotonically increasing or decreasing in beamforming vectors
  - Therefore: **Monotonic problem**

- Can There Be a Magic Algorithm?
  - No, provably NP-hard (Non-deterministic Polynomial-time hard)
  - Exponential complexity but in which parameters?

**Monotonic Optimization Algorithms**

- Improve Lower/upper bounds on optimum: $f_{\text{min}} \leq f_{\text{opt}} \leq f_{\text{max}}$
- Continue until $f_{\text{max}} - f_{\text{min}} < \varepsilon$

Complexity Example 3: Weighted Sum Performance (3)

Branch-Reduce-Bound (BRB) Algorithm

- Iterative weighted max-min fairness
- Global convergence
- Accuracy $\varepsilon > 0$ in finitely many iterations
- Exponential complexity only in #users ($K_r$)
- Polynomial complexity in other parameters (#antennas, #constraints)
Summary: Complexity of Resource Management Problems

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>Zero Forcing</th>
<th>Single Antenna</th>
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<tbody>
<tr>
<td>Sum Performance</td>
<td>NP-hard</td>
<td>Convex</td>
<td>NP-hard</td>
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<td>Max-Min Fairness</td>
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<td>Linear</td>
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<td>NP-hard</td>
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<td>Convex</td>
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<tr>
<td>Harmonic Mean</td>
<td>NP-hard</td>
<td>Convex</td>
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• Recall: The SINR constraints are complicating factor

\[
|h_k^H C_k D_k v_k|^2 \geq \gamma_k \left( \sigma_k^2 + \sum_{i \neq k} |h_k^H C_k D_i v_i|^2 \right)
\]

Signal       SINR       Interference

Three conditions that simplify:
1. Fixed SINRs \( \gamma_k \) (“easy” problem)
2. Allow no interference (called: zero-forcing)
3. Multiplication \( \rightarrow \) Addition (change of variable, single antenna BSs)
Summary: Complexity of Resource Management (2)

• Recall: All Utility Functions are Subjective
  - Pragmatic approach: Select to enable efficient optimization

• Good Choice: Any Problem with Polynomial complexity
  - Example: Weighted max-min fairness
  - Use weights to adapt to other system needs

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• Bad Choice: Weighted Sum Performance
  - Generally NP-hard: Exponential complexity (in #users)
  - Should be avoided – Sometimes needed (virtual queuing techniques)
Summary: Complexity of Resource Management (3)

- Complexity Analysis for Any Dynamic Cooperation Clusters
  - Same optimization algorithms!
  - Extra characteristics can sometime simplify
  - Multi-antenna transmission: Higher complexity, higher performance
Section: Subjective Resource Management

Questions?
Section

Structural Insights
Structure of Optimal Beamforming

- $K_rN$ Complex Optimization Variables: Beamforming vectors $v_1, \ldots, v_{K_r}$
  - Assume: $C_k = D_k = I_N$ and total power constraint ($L = 1$)
  - Only need $K_r-1$ positive parameters

- Any Resource Management Problem is Solved by

\[
\begin{align*}
\mathbf{v}_k^* &= \sqrt{p_k} \left( \mathbf{I}_N + \sum_{i=1}^{K_r} \frac{\lambda_i}{\sigma^2} \mathbf{h}_i \mathbf{h}_i^H \right)^{-1} \mathbf{h}_k \\
&= \text{beamforming power} \quad \text{= beamforming direction}
\end{align*}
\]

- Priority of User $k$: $\lambda_k \leftarrow$ Lagrange multipliers of “Easy” problem

Structure of Optimal Beamforming (2)

- **Geometric Interpretation:**
  - Maximum ratio transmission (MRT)
  - Zero-forcing beamforming (ZFBF)

- **Heuristic Parameter Selection**
  - Make all equal: \( \lambda_1 = \cdots = \lambda_{K_r} = \frac{1}{K_r} \)
  - Known to work remarkably well
  - Many Examples (since 1995): Transmit Wiener filter, Regularized Zero-forcing, Signal-to-leakage beamforming, Virtual SINR beamforming, etc.

- **Tradeoff**
  - Maximize signal vs. minimize interference
  - Selfishness vs. altruism
  - Hard to find optimal tradeoff
  - \( K_r = 2 \): Simple special case
Structure of Optimal Beamforming (3)

- **Example:**
  - One BS with 4 antennas
  - 4 single-antenna users
  - Maximize sum information rate

- **Beamforming Schemes**
  - Optimal beamforming (BRB algorithm)
  - Transmit MMSE filter (regularized ZFBF)
  - ZFBF
  - MRT

**Observations**
- MRT good at low SNR
- ZFBF good at high SNR
- Transmit MMSE always good
Extension: Any Dynamic Cooperation Clusters

• Any Resource Management Problem is Solved by

\[ v_{k}^{(Optimal)} = \sqrt{p_k} \left( \sum_{l=1}^{L} \frac{\mu_l}{q_l} Q_{lk} + \sum_{i=1}^{K_r} \frac{\lambda_i}{\sigma_i^2} D_k^H C_i^H h_i^H h_i^H C_i D_k \right)^{-1} D_k^H C_k^H h_k \]

Power Allocation

\[
\begin{bmatrix}
    p_1 & \cdots & p_{K_r}
\end{bmatrix} = \begin{bmatrix}
    \gamma_1 \sigma_1^2 & \cdots & \gamma_{K_r} \sigma_{K_r}^2
\end{bmatrix} M^\dagger, \quad [M]_{ik} = \begin{cases}
    |h_i^H C_i D_i v_i|^2, & i = k, \\
    -\gamma_k |h_k^H C_k D_i v_i|^2, & i \neq k,
\end{cases}
\]

\[ \gamma_k = \frac{\lambda_k}{\sigma_k^2} h_k^H D_k \left( \sum_{l=1}^{L} \frac{\mu_l}{q_l} Q_{lk} + \sum_{i \neq k} \frac{\lambda_i}{\sigma_i^2} D_k^H C_i^H h_i^H h_i^H C_i D_k \right)^{-1} D_k^H h_k \]

• Parameters

- Priority of User \( k \): \( \lambda_k \)
- Impact of Constraint \( l \): \( \mu_l \)
- Total: \( K_r + L - 2 \) positive parameters (due to sum constraints)
Section: Structural Insights

Questions?
Section

Multi-Objective Network Optimization
Many Network Performance Metrics

- How to Improve Network Performance?
  - Higher user rates
  - Balance user satisfaction/fairness

Other possible metrics:
- Higher total area rate
- More active users (per area)
- Higher energy efficiency

Multi-Objective Optimization
Study conflict/alignment of any metrics
Common tool in engineering/economics

Considered so far
Can be any performance metrics
Basic Properties

- **Multiple Resources**
  - Resource bundle $\mathcal{X}$
  - Feasible resource management: $\mathbf{x} \in \mathcal{X}$

- **Multiple Objectives**
  - Performance metrics: $g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_M(\mathbf{x})$
  - Subjective utility: $f(\cdot)$

- **Performance Region**
  - Compact set
  - Can be non-normal
  - Can be non-convex

Mathematically:

$$\max_{\mathbf{x}} \ f(g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_M(\mathbf{x}))$$

subject to $\mathbf{x} \in \mathcal{X}$
Visualization

- Performance Region is Generally Unknown
  - Sample points can be computed numerically
  - Helps to visualize tradeoffs/possibilities
  - Compare 2-3 metrics

**Algorithm**

Maximize sequence of $f()$

Points out different directions from the origin

*A Posteriori Approach*

Look at region at select operating point
Example: Optimization of Future Networks

- **Design Cellular Network**
  - Symmetric system
  - Select:
    - \( N = \# \text{ BS antennas} \)
    - \( K = \# \text{ users} \)
    - \( P = \text{power/antenna} \)

- **Resource bundle:**

\[
\mathcal{X} = \left\{ [K \ N \ P]^T : \begin{align*}
1 & \leq K \leq \frac{N}{2}, \\
2 & \leq N \leq N_{\text{max}}, \\
0 & \leq P \leq NP_{\text{max}}
\end{align*} \right\}
\]
Example: Optimization of Future Networks (2)

• Coordinated beamforming
  - Each BS serves only its own $K$ users
  - Channels fixed for $T$ channel uses
  - BS knows channels within the cell (cost: $K/T$)
  - Zero-forcing beamforming: no intra-cell interference
  - Interference leaks between cells

• Average User Rate

\[ R_{\text{average}} = B \left( 1 - \frac{K}{T} \right) \log_2 \left( 1 + \frac{P}{K} (N - K) \frac{1}{\sigma^2 \Lambda_1 + P \Lambda_2} \right) \]

- Bandwidth (10 MHz)
- CSI estimation overhead ($T = 1000$)
- Power/user
- Array gain
- Noise \cdot pathloss (1.72 \cdot 10^{-4})
- Relative intercell interference (0.54)
Example: Optimization of Future Networks (3)

- What Consumes Power?
  - Transmit power (+ losses in amplifiers)
  - Circuits attached to each antenna
  - Baseband computations
  - Coding/decoding
  - Backhaul/control signaling, cooling etc.

- Total Power Consumption

\[
P_{\text{total}} = \frac{P}{\eta} + NC_N + KC_K + \frac{C_{\text{precoding}}}{L} + C_0
\]

- Amplifier efficiency (0.31)
- Circuit power per antenna (1 W)
- Circuit power per user (0.3 W)
- Computing zero-forcing (2.3 \times 10^{-6} \cdot NK^2)
- Fixed power (10 W)
Example: Results

3 Objectives

1. Average user rate
   \[ g_1(x) = R_{\text{average}} \] [bit/s/user]

2. Total area rate
   \[ g_2(x) = \frac{K}{A} R_{\text{average}} \] [bit/s/km²]

3. Energy efficiency
   \[ g_3(x) = \frac{K R_{\text{average}}}{P_{\text{total}}} \] [bit/J]

Observations

Area and user rates are conflicting objectives

Only energy efficient at high area rates

Different number of users
Example: Results (2)

- Energy Efficiency vs. User Rates
  - Utility functions normalized by utopia point

Observations

Aligned for small user rates

Conflicting for high user rates
Summary

- **Multi-Objective Optimization**
  - Rigorous way to study problems with multiple performance metrics
  - Metrics are generally conflicting, but can be aligned

- **Network Design and Optimization**
  - Metrics can be: User rates, total area rates, energy-efficiency, etc.
  - A posteriori approach: Visualize tradeoffs → Make decision
  - Open problems: How to use framework for design of future networks?


Section: Multi-Objective Network Optimization

Questions?
Summary: Part 1
Summary: Part 1

- Resource Management in Multi-Antenna Multi-Cell Systems
  - Divide power between users and spatial directions
  - Solve a multi-objective optimization problem
  - Pareto boundary: Set of efficient solutions

- Subjective Utility Function
  - Selection has fundamental impact on solvability
  - Multi-antenna transmission: More possibilities – higher complexity
  - Pragmatic approach: Select to enable efficient optimization
  - Polynomial complexity: Weighted max-min fairness etc.
  - Not solvable in practice: Weighted sum performance etc.

- Multi-Objective Network Optimization
  - Practical optimization problems may have more than one metric
  - General framework: Study tradeoff between metrics
  - A posterior approach: Visualize the tradeoff and make decisions
Coffee Break

• Thank you for listening!
  - Questions?

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After the Break: Applications for Future Networks
  - Energy-Efficiency in Multi-User Multi-Hop Networks
  - Distributed Implementation and Spectrum Sharing
  - Resource Management with Confidentiality Constraints